A simplistic Sagnac ring

Two continuous monochromatic light beams are sent in opposite directions around a circular loop of radius r. The light beams are emitted from a common source, and are therefore in phase at the emitter. The light beams will meet each other back at the source, and the phase difference between them is measured. The ring is rotating around its centre with a peripheral speed v.

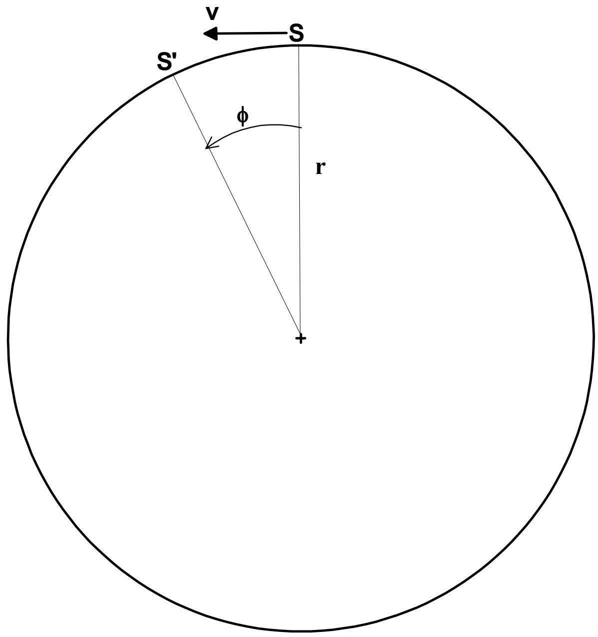

Fig. 1

We will calculate what the Special Theory of Relativity and the Ritz Emission Theory predict the phase difference will be.

There are two ways of calculating the phase difference:

1. The difference in transit time for the two beams can be calculated. The phase difference is then $\Delta\varphi = \left(2\pi c/\lambda\right)\cdot\Delta t$. The transit time is the time a plane of equal phase uses to move around the ring.
2. The number of wavelengths in the two beams can be compared. The phase difference is then $\Delta\varphi = 2\pi\cdot\Delta N$ where ΔN is the difference in the number of wavelengths in the two beams.

Even if the two methods necessarily must give the same result, we will calculate the predictions both ways.

In all cases the calculation will be made in the non rotating inertial frame where the centre of the ring is stationary. All speeds and wavelengths will in the following be referred to this frame of reference.

The prediction of the Special Theory of Relativity

<u>The difference in transit times method</u>.

According to the Special Theory of Relativity the speed of light is c for both beams.

The transit time t_f of the beam emitted in the forward direction:

$$(2\pi + \phi) r = c \cdot t_f$$
$$\phi = \tfrac{v}{r} \cdot t_f$$
$$t_f = \frac{2\pi r}{c - v}$$

The transit time t_b of the beam emitted in the reverse direction:

$$(2\pi - \phi) r = c \cdot t_b$$
$$\phi = \tfrac{v}{r} \cdot t_b$$
$$t_b = \frac{2\pi r}{c + v}$$

The difference in transit time Δt:

$$\Delta t = t_f - t_b = \frac{2\pi r}{c - v} - \frac{2\pi r}{c + v} = \frac{4\pi r v}{c^2\left(1 - \left(\tfrac{v}{c}\right)^2\right)}$$

A first order approximation in v/c:

$$\Delta t \approx \frac{4\pi r v}{c^2}$$

Inserting the area of the ring $A = \pi r^2$ and the angular velocity of the ring $\omega = v/r$ yields:

$$\Delta t \approx \frac{4A\omega}{c^2}$$

The predicted phase difference is thus:

$$\Delta\varphi \approx \frac{8\pi A w}{\lambda c}$$

This is in accordance with the experimentally verified equation for a Sagnac ring.

The difference in number of wavelengths method

Since the source is moving with the speed v, the wavelengths of the beams will according to the Special Theory of Relativity be Doppler shifted.

Forward beam:

$$\lambda_f = \sqrt{\frac{c-v}{c+v}} \cdot \lambda \approx \left(1 - \frac{v}{c}\right) \cdot \lambda$$

$$N_f = \frac{2\pi r}{\lambda_f} \approx \frac{2\pi r}{\left(1-\frac{v}{c}\right)\cdot \lambda}$$ where N_f is the number of wavelengths in the forward beam

Backward beam:

$$\lambda_b = \sqrt{\frac{c+v}{c-v}} \cdot \lambda \approx \left(1 + \frac{v}{c}\right) \cdot \lambda$$

$$N_b = \frac{2\pi r}{\lambda_b} \approx \frac{2\pi r}{\left(1+\frac{v}{c}\right)\cdot \lambda}$$ where N_b is the number of wavelengths in the backward beam

$$\Delta N = N_f - N_b = \frac{2\pi r}{\left(1-\frac{v}{c}\right)\cdot \lambda} - \frac{2\pi r}{\left(1+\frac{v}{c}\right)\cdot \lambda} = \frac{4\pi r v}{\lambda c\left(1-\left(\frac{v}{c}\right)^2\right)} \approx \frac{4\pi r v}{\lambda c}$$

Inserting the area of the ring $A = \pi r^2$ and the angular velocity of the ring $\omega = v/r$ yields:

$$\Delta N \approx \frac{4A\omega}{\lambda c}$$

The predicted phase difference is thus:

$$\Delta\varphi \approx \frac{8\pi A w}{\lambda c}$$

Both methods give the same result, as they should.

The prediction of the Ritz Emission Theory

<u>The difference in transit times method</u>.

According to the Ritz Emission Theory the speed of light is c+v in the forward beam and c-v in the backward beam.

The transit time t_f of the beam emitted in the forward direction:

$$(2\pi + \phi) r = (c + v) \cdot t_f$$

$$\phi = \tfrac{v}{r} \cdot t_f$$

$$t_f = \frac{2\pi r}{c}$$

The transit time t_b of the beam emitted in the reverse direction:

$$(2\pi - \phi) r = (c - v) \cdot t_b$$

$$\phi = \tfrac{v}{r} \cdot t_b$$

$$t_b = \frac{2\pi r}{c}$$

The difference in transit time Δt:

$$\Delta t = t_f - t_b = \frac{2\pi r}{c} - \frac{2\pi r}{c} = 0$$

The predicted phase difference is thus: $\Delta\varphi = 0$

<u>The difference in number of wavelengths method</u>

According to Ritz Emission theory, wavelengths are not Doppler shifted. (Because of the Galilean transform.)

Forward beam:

$$\lambda_f = \lambda$$

$$N_f = \frac{2\pi r}{\lambda}$$ where N_f is the number of wavelengths in the forward beam

Backward beam:

$$\lambda_b = \lambda$$

$$N_b = \frac{2\pi r}{\lambda}$$ where N_b is the number of wavelengths in the backward beam

$$\Delta N = N_f - N_b = \frac{2\pi r}{\lambda} - \frac{2\pi r}{\lambda} = 0$$

The predicted phase difference is thus: $\Delta\varphi = 0$

Since the experimentally verified equation for a Sagnac ring is $\Delta\varphi \approx 8\pi Aw/\lambda c$, the Sagnac experiment falsifies the Ritz Emission Theory.