The prediction of fringe shifts in Michelson & Morley's repetition of Fizeau's experiment calculated by comparing number of wavelengths

- Let ν be the frequency of the light source as measured in the lab frame.
- Let λ be the wavelength in vacuum of light with frequency ν, $\lambda = c/\nu$
- Let λ_f be the wavelength of the ray that is moving with the water, as measured in the lab frame.
- Let λ_b be the wavelength of the ray that is moving in the opposite direction, as measured in the lab frame.
- Let $c_f = \frac{c}{n} + v \cdot x$ be the speed of the light in the ray that is moving with the water, as measured in the lab frame.
- Let $c_b = \frac{c}{n} - v \cdot x$ be the speed of the light in the ray that is moving in the opposite direction, as measured in the lab frame.

The wavelength of the beam going with the water will be Doppler shifted in the lab frame:

$$\lambda_f = \frac{c_f}{\nu} = \frac{\lambda}{c} \cdot c_f = \frac{\lambda}{c} \cdot \left(\frac{c}{n} + v \cdot x \right)$$

The wavelength of the beam going in the opposite direction will be Doppler shifted in the lab frame:

$$\lambda_b = \frac{c_b}{\nu} = \frac{\lambda}{c} \cdot c_b = \frac{\lambda}{c} \cdot \left(\frac{c}{n} - v \cdot x \right)$$

The number of wavelengths in the beams will be:

$$N_f = \frac{L}{\lambda_f} = \frac{L}{\frac{\lambda}{c}\left(\frac{c}{n} + v \cdot x \right)} = \frac{Lc}{\lambda} \cdot \frac{1}{\left(\frac{c}{n} + v \cdot x \right)}$$

$$N_b = \frac{L}{\lambda_b} = \frac{L}{\frac{\lambda}{c}\left(\frac{c}{n} - v \cdot x \right)} = \frac{Lc}{\lambda} \cdot \frac{1}{\left(\frac{c}{n} - v \cdot x \right)}$$

The difference in the number of wavelengths will be:

$$\Delta N = N_b - N_f = \frac{Lc}{\lambda} \cdot \frac{1}{\left(\frac{c}{n} - v \cdot x \right)} - \frac{Lc}{\lambda} \cdot \frac{1}{\left(\frac{c}{n} + v \cdot x \right)} = \frac{2Lvn^2 \cdot x}{\lambda c \left(1 - \left(\frac{nv}{c} \right)^2 \right)} \approx \frac{2Lvn^2 \cdot x}{\lambda c}$$

Michelson measured the fringe shift when the water flow was reversed.

So the total fringe shift will be $2 \cdot \Delta N = \frac{4Lvn^2 \cdot x}{\lambda c}$

According to SR, the speed of light in the moving water is: $c_{f,b} \approx \frac{c}{n} \pm v \cdot \left(1 - \frac{1}{n^2}\right)$,

$x = \left(1 - \frac{1}{n^2}\right)$

Fringe shift = $\frac{4Lv(n^2-1)}{\lambda c}$, which is the same calculated by transit time method.

According to a theory which say the speed of light in the moving water is: $c_{f,b} = \frac{c}{n} \pm v$, $x = 1$

Fringe shift = $\frac{4Lvn^2}{\lambda c}$, which is the same calculated by transit time method.